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A Study on Modeling Financial Mathematics by the Computational Program and Its Applications

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ABSTRACT

Financial mathematics plays a pivotal role in various aspects of modern economics and finance. This paper provides an introduction to the fundamental concepts, theories, and applications of financial mathematics. It begins by outlining the basic principles of financial mathematics, including the time value of money, interest rates, and compounding. Computational programs enhance these mathematical models, offering robust solutions and efficient computation for complex financial problems. This study explores the integration of computational programs with financial mathematics, their methodologies, and applications in the finance sector. The results underscore the significance of computational methods in improving the accuracy, speed, and scalability of financial models, ultimately contributing to better decision-making and risk management. We explore fundamental concepts, models, and techniques employed in financial mathematics, aiming to provide a comprehensive understanding of their applications and significance in real-world financial scenarios. This paper provides a comprehensive overview of the application of differential equations in financial mathematics, highlighting key models such as the Black-Scholes model, interest rate models, and optimal investment strategies.

Keywords: Financial mathematics, Interest models, Hamilton-Jacobi-Bellman equation, and Fortran program.

INTRODUCTION:

Financial mathematics, also known as mathematical finance or quantitative finance, is an interdisciplinary field that applies mathematical techniques to solve problems in finance. It encompasses a wide range of topics, including investments, risk management, derivatives pricing, and portfolio optimization. The primary objective of financial mathematics is to provide quantitative tools and models for analyzing and managing financial risk and uncertainty. Financial mathematics, often referred to as mathematical finance, is an interdisciplinary field that utilizes mathematical techniques and models to analyze financial markets, instruments, and strategies (Black and Scholes, 1973). The application of mathematical methodologies in finance enables practitioners to make informed decisions regarding investments, risk management, pricing of derivatives, and portfolio optimization. This article aims to elucidate the mathematical foundations of financial mathematics and their practical implications in the realm of finance (Merton, 1973). Financial mathematics integrates mathematical theories with financial practice, providing essential tools for valuing financial instruments, managing risks, and making informed investment decisions. Among these mathematical tools, differential equations are particularly significant. They help in modeling the temporal evolution of various financial variables and in deriving pricing formulas for complex financial derivatives (Cox *et al*., 1985). Time Value of Money: The concept that money available today is worth more than the same amount in the future due to its potential earning capacity. The rate at which interest is paid or received on an investment or loan, expressed as a percentage.

The process by which an investment earns interest not only on the initial principal but also on the accumulated interest from previous periods (Glarsserman, 2004). Financial mathematics, a field that sits at the intersection of mathematics, finance, and economics, is integral to understanding and navigating the complexities of modern financial systems. It involves the application of mathematical methods and models to solve problems related to financial markets, including pricing of derivatives, risk management, and portfolio optimization (Von *et al*., 2011). This field has grown significantly for a long time, driven by the increasing complexity of financial instruments and the need for sophisticated quantitative techniques to manage risk and return. At its core, financial mathematics provides the foundational tools necessary for the valuation of financial assets and the assessment of market risks (Von *et al*., 2015 and Zinman, 2015). These tools include stochastic calculus, probability theory, and statistical methods, which model the random behavior of asset prices and interest rates. The evolution of these models has been crucial for the development of modern finance, enabling practitioners to predict market trends, hedge against potential losses, and devise strategic investment plans (Agarwal and Mazumder, 2013). The practical applications of financial mathematics are vast and varied. From the Black-Scholes model for option pricing to the Capital Asset Pricing Model (CAPM) for determining the expected return on assets, financial mathematics offers essential frameworks that underpin decision-making in finance (Cole *et al*., 2011; Gathergood, 2012). Moreover, the advent of computational techniques (Cornelis and Lech, 2022) and the availability of large datasets have further enhanced the precision and applicability of financial models, making them indispensable tools in the toolkit of financial analysts and economists (Geradi and Meier, 2013; Sami *et al.,* 2021).

This text aims to delve into the foundations of financial mathematics, exploring the theoretical underpinnings and practical applications that make this field so vital. We will cover fundamental concepts such as arbitrage theory, stochastic processes, and risk-neutral valuation, providing a comprehensive understanding of how these principles are applied in real-world financial scenarios. By bridging the gap between theory and practice, this exploration will equip readers with the knowledge and skills necessary to tackle complex financial challenges and innovate within the ever-evolving landscape of global finance (Gibson *et al.,* 2014 and Sayingzoga *et al*., 2016).

METHOLODOGY:

The study employs a multi-faceted approach, including a literature review, case studies, and computational experiments.

Review of Literature

We review existing research on financial mathematics and computational finance, focusing on key models and their computational implementations.

Case Studies

We analyze case studies where computational programs have been successfully applied to solve financial problems.

Computational Experiments

We implement and test various financial models using computational programs to evaluate their performance and accuracy. Modeling financial mathematics involves a rigorous and systematic approach to understanding the quantitative dynamics of financial markets. The mathematical methodology employed in this field encompasses several key components, each building upon fundamental principles of mathematics and statistics. Here, we outline the primary elements of this methodology:

Step I: Stochastic Processes and Brownian Motion

Financial markets are inherently random, and stochastic processes are used to model this randomness. This system is a sum of various-parameters indexed by time, implying the evolution of a system over time. Key Concept: $X(t)$ where t is time and $X(t)$represents the state of the process at time t. A central model in financial mathematics is the Brownian motion (or Wiener process), denoted as $W(t)$. Continuous paths, stationarity, and independence of increments characterize it. $W(t)$ is a process such that

$W(0) = 0, W(t) - W(s) \sim N(0, t-s)$ for $0 \leq s < t$, and $W(t)$ has independent increments.

Step II: Stochastic for Differential Equations (SDEs)
SDEs describe the dynamics of asset prices and interest rates. They combine deterministic and stochastic components to model continuous-time processes in which the general form given by

$$dX(t) = \mu(X(t),t)dt + \sigma(X(t),t)dW(t)dX(t) = \mu(X(t),t)dt + \sigma(X(t),t)dW(t), \quad (1)$$

where $\mu(X(t),t)$ implies the drift term representing the deterministic part and $\sigma(X(t),t)$ implies the diffusion term representing the random part.

Step III: Applying the Itô calculus

Itô calculus extends traditional calculus to stochastic processes. Itô's lemma, a fundamental result, provides the differential of a function of a stochastic process. Itô's Lemma: For $X(t)$ following $dX(t) = \mu dt + \sigma dW(t)$ and a twice differentiable function $f(X,t)$,

$$df(X,t) = (\frac{\partial f}{\partial t} + \mu\frac{\partial f}{\partial X} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial X^2})dt + \partial X2) + \sigma\frac{\partial f}{\partial X}dW(t). \quad (2)$$

Step IV: Calculate the risk-neutral valuation
In financial mathematics, the concept of a risk-neutral measure is used for pricing derivatives. Under the risk-neutral measure, the discounted price of a financial asset is a martingale. Fundamental Theorem of Asset Pricing: States that a market is arbitrage-free if and only if there exists a risk-neutral measure $\overline{\mathbb{Q}}$ such that the discounted price process is a martingale.

Step V: Using partial Differential Equations (PDEs)
PDEs arise in the pricing of derivative securities. The Black-Scholes equation, for example, is a PDE that governs the price of European options. Black-Scholes Equation: Derived using Itô's lemma and the principle of no-arbitrage,

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial t} - rV = 0, \quad (3)$$

Where V is the option price, S is the underlying asset price, σ is the volatility, and r is the risk-free rate.

Step VI: Using Numerical Methods
Analytical solutions to SDEs and PDEs are often not possible for complex models. Numerical methods, such as finite difference methods, Monte Carlo simulations, and binomial/trinomial trees, are employed to approximate solutions. The Monte Carlo Simulation: Uses random sampling to estimate the expected value of complex financial instruments,

$$\hat{V} = \frac{1}{N}\sum_{i=1}^{N} V_i, \quad (4)$$

Where V_i are the simulated payoffs, and N is the number of simulations

Step vii: Applying the Optimization Techniques
Portfolio optimization and risk management often involve solving optimization problems. Techniques such as quadratic programming and dynamic programming are used to determine optimal asset allocations. Markowitz Portfolio Optimization: Minimizes portfolio variance subject to a given expected return,

$\min_w \boldsymbol{w}^{\boldsymbol{T}} \sum \boldsymbol{w}$ subject to $\boldsymbol{w}^{\boldsymbol{T}}\mu = \mu_p, \boldsymbol{w}^T 1 = 1.$

Where **w** is the vector of asset weights, Σ is the covariance matrix, μ is the vector of expected returns, and μ_p is the target portfolio return.
These methodologies form the backbone of financial mathematics, enabling practitioners to develop robust models for pricing, hedging, and optimizing financial instruments. By integrating these mathematical techniques, we can better understand and predict the behavior of financial markets, thereby enhancing decision-making and strategic planning in finance.

Creating Mathematical Model

Deterministic Differential Equations:

Deterministic models describe systems that evolve over time with no randomness. For instance, the compound interest model can be expressed as an ordinary differential equation (ODE).

$$\frac{dp(t)}{dt} = rp(t), \quad (5)$$

Where $P(t)$ is the price at time t and r is the interest rate.

Black-Scholes Model

The Black-Scholes model is a seminal work in financial mathematics, providing a closed-form solution for pricing European call and put options. The model assumes that the price of the underlying asset follows a geometric Brownian motion,

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t). \quad (6)$$

Interest Rate Models

Interest rate models describe the evolution of interest rates over time. Common models include: Vasicek Model:

$$dr(t) = a(b - r(t))dt + \sigma dW(t). \quad (7)$$

This mean-reverting model assumes that the interest rate $r(t)$ tends to revert to a long-term mean b with speed a.

Cox-Ingersoll-Ross (CIR) Model

$$dr(t) = a(b - r(t))dt + \sigma\sqrt{r(t)}dW(t). \quad (8)$$

The CIR model ensures that interest rates remain positive by incorporating a square-root diffusion term.

Optimal Investment Strategies

Optimal investment problems involve determining the best portfolio allocation to maximize expected returns while minimizing risk. The Hamilton-Jacobi-Bellman (HJB) equation is a key tool in solving these problems. For a portfolio (t) invested in risky asset $S(t)$ and risk-free asset $B(t)$, the wealth $X(t)$ evolves as:

$$dX(t) = \pi(t)dS(t) + (X(t) - \pi(t))rdt. \quad (9)$$

The HJB equation for the value function $V(X, t)$ is:

$$\frac{\partial V}{\partial t} + MaxX_{\pi}[\pi\mu\frac{\partial V}{\partial X}\frac{1}{2}\sigma^2\pi^2\frac{\partial^2 V}{\partial X^2}] + rX\frac{\partial V}{\partial X} = 0, \quad (10)$$

Advanced Topics in Financial Mathematics

Stochastic Processes

Mathematical models that describe the evolution of financial variables over time, taking into account random fluctuations and uncertainty.

Options Pricing Theory

The Black-Scholes model and its extensions are widely used to price options and other derivatives, incorporating factors such as volatility and time to expiration.

Monte Carlo Simulation

A computational technique used to estimate the probability distribution of outcomes in complex financial systems by generating random samples from relevant probability distributions.

Value at Risk (VaR)

A statistical measure of the maximum potential loss that a portfolio may incur over a specified time horizon at a given confidence level, used for risk management purposes.

Challenges and Future Directions

Time Value of Money

The concept that a certain amount of money today has different value than the same amount in the future due to factors like interest rates, inflation, and opportunity costs.

Discounted Cash Flow Analysis

A method used to evaluate the present value of `future cash flows by discounting them at an appropriate interest rate.

Interest Rates and Compounding
Understanding different types of interest rates (simple, compound) and their impact on the valuation of financial instruments.

Models and Techniques in Financial Mathematics

Black-Scholes Model
A mathematical model used to determine the theoretical price of European-style options by considering factors such as underlying asset price, volatility, time to expiration, and risk-free rate.

Binomial Option Pricing Model
A discrete-time model for valuing options that breaks down the time to expiration into a number of intervals and calculates the option price at each interval.

Capital Asset Pricing Model (CAPM)
A model used to determine the expected return of an asset based on its risk as measured by beta, the risk-free rate, and the expected market return.

Portfolio Optimization
Mathematical techniques such as mean-variance analysis and modern portfolio theory are used to construct portfolios that maximize return for a given level of risk or minimize risk for a given level of return.

Risk Management
Techniques like value-at-risk (VaR) and the stress testing employ mathematical models to quantify and manage financial risk by estimating the potential losses under adverse market conditions.

Monte Carlo Simulation
A probabilistic technique used to model the behavior of financial instruments and portfolios by generating random variables and simulating various market scenarios.

Applications of Financial Mathematics
Computational programs are essential in risk management for calculating Value at Risk (VaR), stress testing, and scenario analysis. They enable the assessment of potential losses under various scenarios and help in making informed risk management decisions including, Algorithmic Trading: Algorithmic trading relies heavily on computational programs to execute trades based on predefined criteria. These programs analyze data in the market, tracking trading opportunities, and execute trades at high speeds, optimizing trading strategies, Portfolio Optimization: Computational programs assist in portfolio optimization by solving complex optimization problems. They help in determining the optimal asset allocation to maximize returns for a given level of risk. Techniques like mean-variance optimization, robust optimization, and machine learning algorithms are commonly used.

Option Pricing and Hedging
Financial institutions and investors use mathematical models to price options accurately and develop hedging strategies to mitigate risk.

Asset Allocation
Pension funds, endowments, and individual investors utilize mathematical techniques to allocate assets across different classes to achieve their investment objectives.

Risk Management
Banks, insurance companies, and hedge funds employ mathematical models to assess and manage various types of risk, including market risk, credit risk, and operational risk.

Investment Analysis
Financial mathematics is used to evaluate the potential returns and the risks associated with various investment opportunities, helping investors make informed decisions.

Risk Management
Financial institutions employ mathematical models to quantify and manage risks, such as market risk, credit risk, and operational risk.

Derivatives Pricing
Complex financial instruments, such as options and futures, are valued using mathematical models derived from principles of stochastic calculus and probability theory.

Portfolio Optimization
Financial mathematics enables investors to construct optimal portfolios that balance risk

and return by diversifying investments across different asset classes.

Optimal Interest

Basically, optimal interest is known as the opportunity cost. This is classified in the followings,

Interest in Mathematics

It is thought that Jacob Bernoulli discovered the mathematical constant e by studying a question about compound interest. He realized that if an

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e,$$

Where n is the number of times the interest is to be compounded in a year.

Earning Interest

When you lend money, you often earn interest. For example, depositing money into a bank account is like lending money to the bank. They'll use your cash to make loans (charging borrowers more interest than they pay you). The bank pays you interest when you use savings accounts and certificates of deposit (CDs) because they want you to keep your money at the bank. Unless you have something better to do with the funds, you'll leave it in the bank to earn interest.

Interest Rate

The fee charged by a lender to a borrower for the use of borrowed money is usually expressed as an annual percentage of the principal; the rate is dependent upon the time value of money, the credit risk of the borrower, and the inflation rate. Here, interest per year is divided by the principal amount, expressed as a percentage. Also called interest rate.

account that starts with $\$1.00$ and pays say 100% interest per year, at the end of the year, the value is $\$2.00$; but if the interest is computed and added twice in the year, the $\$1$ is multiplied by 1.5 twice, yielding $\$1.00\times 1.5^2 = \$2.25$. Compounding quarterly yields

$\$1.00\times 1.25^4 = \$2.4414\ldots$, and so on.

Bernoulli modeled this as follows:

(11)

A rate which is charged or paid for the use of money. An interest rate is often expressed as an annual percentage of the principal. It is calculated by dividing the amount of interest by the amount of principal. Interest rates often change as a result of inflation and Federal Reserve Board policies.

For example, if a lender (such as a bank) charges a customer $\$90$ in a year on a loan of $\$1000$, then the interest rate would be

$$\frac{90}{1000}\times 100\% = 9\%.$$

When the interest is paid, for example, for a credit card, a mortgage, or a loan, the interest rate is expressed as annual percentage rate

Simple Interest

Simple interest is the interest computed on the principle for the entire period it is borrowed. If a principle of Tk. P is borrowed at a simple interest of $r\%$ per year for a period of t years, then the simple interest is determined by the formula:

$$I = Prt = \text{Principle}\times\text{Rate}\times\text{Time}.$$

Thus, the amount A due at the end of period of t years is,

$$A = \text{Principle} + \text{Interest} = P + Prt = P(1+rt). \quad (12)$$

$$\text{Therefore, } P = \frac{A}{1+rt}. \quad (13)$$

Simple interest is charged as yearly basis. When the time is given in months, then

$$t = \frac{\text{number of months}}{12}\ years$$

When time is given in weeks, then

$$t = \frac{\text{number of weeks}}{52}\ years$$

When time is given in days, then

$$t = \frac{\text{number of days}}{360} \text{ or } \frac{\text{number of days}}{365} \text{ years}$$

Simple interest is called simple because it ignores the effects of compounding.

Graphical Representation of Simple Interest

Simple interest is the most basic type of return. Depositing Tk.100 into an account with 50% simple (annual) interest looks like this:

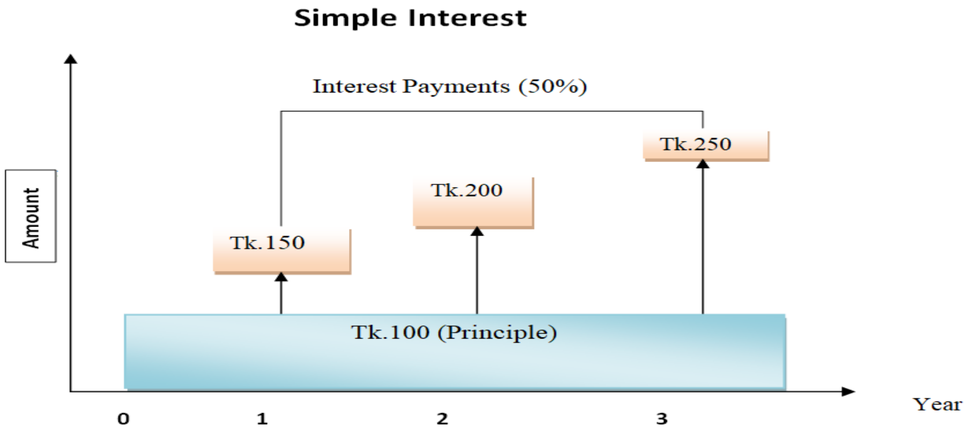

Fig. 1: Diagram of Simple Interest.

I start with a principal of $Tk.100$ and earn $Tk.50$ each year.

Exact Simple Interest

When the time is given in days, we calculate exact simple interest on the basis of 365-day year, that is

$$t = \frac{\text{number of days}}{365}.$$

Exact time is found as the exact number of days including all days except the first. Exact time may be obtained as the difference between serial numbers of the given dates. In leap years, the serial number of the day is increased by for all dates after February 28.

Ordinary Simple Interest

When we calculate interest on the basis of a 360-day year, that is known as ordinary simple interest (Banker's rule), that is

$$t = \frac{\text{number of days}}{360}.$$

Approximate time is found by assuming that each month has 30 days.

The time between dates

There are four methods for computing simple interest between dates:

(1) Exact time and ordinary interest (the Banker's rule) ;
(2) Exact time and exact interest ;
(3) Approximate time and ordinary interest ;
(4) Approximate time and exact interest.

The Banker's rule is the common method in the United States and international business transact-tions; the general practice in Canada is to use Method 2. Methods 3 and 4 are used very rarely.

Problem: On January 10, Mr. A borrows $1000 on a demand loan from his bank. Interest is paid at the end of each quarter (March 31, June 30, September 30, and December 31) and at the time of last payment. Interest is calculated at a rate of 12% on the balance of the loan outstanding. Mr. A repaid the loan with the following payments:

March 1	$ 100
April 17	$ 300
July 12	$ 200
August 20	$ 100
October 18	$ 300
Total	$ 1000

Calculate the interest payments required and the total interest paid

Solution

Dates	No. of Days	Balance	Interest (*I=Prt*)
Jan. 10 - Mar. 1	50	1000	100×0.12 × (50/360) = 16.67
Mar. 1 - Mar.31	30	900	900×0.12 × (30/360) = 9.00
Mar.31 payment= $ 25.67			

Mar.31- Apr.17	17	900	900×0.12×(17/360)= 5.10
Apr.17 - June30	74	600	600×0.12×(74/360)=14.80
June 30 payment=$ 19.90			
June 30-July 12	12	600	600×0.12×(12/360)=2.40
July 12- Aug.20	39	400	400×0.12×(39/360)=5.20
Aug.20-Sep. 30	41	300	300×0.12×(41/360)=4.1
Sep. 30 payment=$ 11.70			
Sep.30- Oct.18	18	300	300×0.12×(18/360)=1.80
Oct 18 payment=$ 1.80			

$\therefore$

Total interest paid = $ (25.67+ 19.90 + 11.70 + 1.80)
= $ 59.07

FORTRAN Program for Solving Simple Interest

Problem: Using FORTRAN program find the accumulated value of sum $ 1000 for 3 years at a rate of 12%. Evaluate Simple interest.

Solution

FORTRAN program code:

```
10 READ*,P,r,t
S=P*(1+r*t)
PRINT 33, S
33 FORMAT (//2X,"S=", F10.2)
GOTO 10
END
```

Input : 1000 0.12 3

Output : $S = 1360.00$

Compound Interest

If the interest on a particular principal sum is added to it after each prefixed period, the whole amount earns interest for the next period, and then the interest calculated in this manner is called compound interest. The period after which interest becomes due is called interest period or conversion period. If P is the principle and r is the interest rate then at the end of one year amount will be

$$A_1 = P(1+r)$$

Similarly,

$$A_2 = A_1 + A_1 r$$
$$= P(1+r)^2$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\therefore A_n = P(1+r)^n . \tag{15}$$

In the compound interest formula the rate of interest, r is given by

$$r = \frac{\text{interest rate per year}}{\text{No. of compounding period per year}} = \frac{i}{t}$$

$$\therefore A_n = P\left(1+\frac{i}{t}\right)^n . \tag{16}$$

Graphical Representation of Compound Interest

Now, reinvesting our interest annually looks like this:

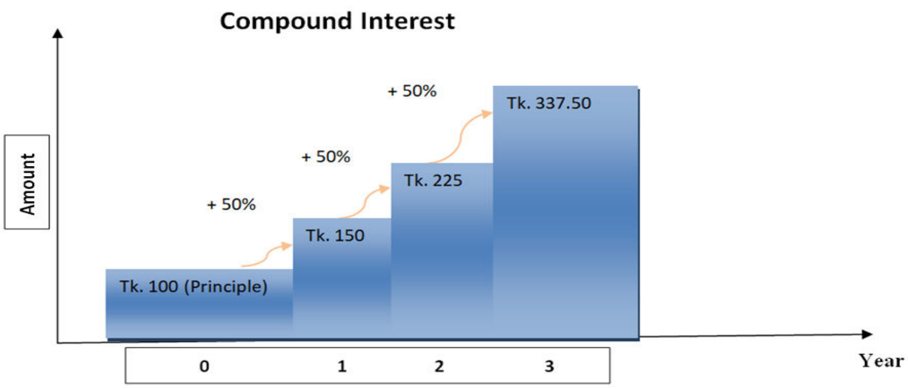

Fig. 2: Diagram of compound interest.

We earn $Tk.50$ from year 0 to 1, just like with simple interest. But in year 1 to 2, now that our total is $Tk.150$, we can earn $Tk.75$ this year $(50\%\times 150)$ giving us $Tk.225$. In year 2 to 3 we have $Tk.225$, so we earn 50% of that, or $Tk.112.50$.

Accumulated value

The interest may be converted into principal annually, semiannually, quarterly, monthly, weekly, daily or continuously. The number of times interest is converted in one year is called the frequency of conversion. The rate of interest is usually stated as an annual interest rate, referred to as the nominal rate of interest. The phrase "interest at 12%" or "money worth 12%" means 12% compounded annually; otherwise, the frequency of conversion is indicated, e.g., 16% compounded semiannually, 10% compounded daily. When compounding daily, most U.S. banks use a 365-day year. We will use the following notations:

$P \equiv$ Original principle or the present value of S.
$S \equiv$ Compound amount of P or accumulated value of P.
$n \equiv$ Total number of interest periods.
$m \equiv$ Number of interest period per year.
$j_m \equiv$ Nominal (yearly) interest rate which is compounded m times per year.
$i \equiv$ Rate of interest.

The interest rate per period, $i = \dfrac{j_m}{m}$. For example $j_{12} = 12\%$ means that a nominal rate of 12% is converted 12 times per year i.e., $i = \dfrac{12\%}{12} = 1\% = 0.01$ being the interest per month.

Let P be the principal, i be the interest per period, then accumulated value.

At end of 1st period $= P + Pi = P(1+i)$

At end of 2nd period

$$= P(1+i)(1+i)i = P(1+i)(1+i) = P(1+i)^2$$

Similarly, at end of nth period $= P(1+i)^n$

$\therefore\ S = P(1+i)^n$ is the fundamental compound interest formula, $(1+i)^n$ is known as accumulation factor. The accumulated value S of principle P at rate j_m for t years is:

$$\begin{aligned} S &= P(1+i)^n \\ &= P\left(1+\frac{j_m}{m}\right)^{mt} \\ &= P\left\{\left(1+\frac{j_m}{m}\right)^{m}\right\}^t \end{aligned}$$

The accumulated value under continuous com-pounding is obtained by letting

$$\begin{aligned} S &= \lim_{m\to\infty} P\left[\left(1+\frac{j_m}{m}\right)^m\right]^t \\ &= P.\lim_{m\to\infty}\left[\left(1+\frac{j_m}{m}\right)^m\right]^t \end{aligned} \qquad (17)$$

We know

$$\lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m = e^x. \qquad (18)$$

$$\therefore\ S = P\left[e^{j_\infty}\right]^t = Pe^{j_\infty t}$$

Problem: Find the accumulated value of sum $ 1000 for 3 years at a rate of 12%

a) Evaluate Simple interest.
b) Evaluate Compound interest 1) annually, 2) Semi-annually

Solution (a): Here, the principal, $P = 1000$, rate of interest, $r = \dfrac{12}{100} = 0.12$, time, $t = 3$.

$$\begin{aligned} S &= P(1+rt) \\ &= 1000(1+0.12\times 3) \\ &= 1360 \end{aligned}$$

Solution (b):

1) Here, principal, $P = 1000$, rate of interest, $i = \frac{12}{100} = 0.12$, time, $n = 3$.

We know for annual compound interest,

$$S = P(1 + i)^n$$
$$= 1000(1 + 0.12)^3$$
$$= 1404.928$$

2) Here, principal, $P = 1000$, rate of interest,

$$i = \frac{j_m}{m} = \frac{12\%}{2}$$
$$= 6\% = 0.06$$

Time, $n = m \times t = 2 \times 3 = 6$

We know for semi-annual compound interest,

$$S = P(1 + i)^n$$
$$= 1000\ (1 + 0.06)^6$$
$$= 1418.519112$$

FORTRAN Program for Solving Compound Interest

Problem: Using FORTRAN program find the accumulated value of sum $ 1000 for 3 years at a rate of 12%.Evaluate Compound interest 1) annually, 2) semi-annually.

Solution:

FORTRAN program code:

```
REAL::i
10 READ*,P,m,t,r
n=m*t
i=r/m
 S=P*(1+i) **n
PRINT 33, S
33 FORMAT (//2X,"S=", F10.2)
   GOTO 10
   END
```

Input: 1000 1 3 0.12

Output: $S = 1404.93$

Input: 1000 2 3 0.12

Output: $S = 1418.52$

CONCLUSION:

Financial mathematics plays a critical role in modern finance by providing quantitative tools and models for analyzing and managing financial risk. By understanding the basic concepts and applications of financial mathematics, practitioners can make more informed decisions and mitigate potential risks in the dynamic and complex world of finance. Differential equations, especially stochastic differential equations, are fundamental in financial mathematics. They provide frameworks for modeling the dynamic behavior of financial variables, pricing derivatives, and optimizing investment strategies. Ongoing research continues to refine these models, incorporating new data and improving computational techniques to better capture market realities. The integration of computational programs with financial mathematics significantly enhances the capability to model, analyze, and solve complex financial problems. The applications in risk management, algorithmic trading, and portfolio optimization illustrate the practical benefits of these technologies. As computational power continues to grow, the future of financial modeling will increasingly rely on sophisticated computational methods, promising more precise and efficient solutions for the finance industry.

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CONFLICTS OF INTEREST:

The authors declare that there is no conflict of interest.

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